and $j = \sqrt{-1}$. The lower limit is 0 rather than $-\infty$ in this case because **Z** represents power and therefore is non-negative.

Assuming the $\{\mathbf{z}_j\}$ are independent and identically-distributed (i.i.d.), (A-2) and (A-9) yield:

$$\Phi_{\mathbf{Z}_K}(\omega) \mid \mathbf{J} = E\left[\exp\left(j\omega \sum_{j=1}^{\mathbf{J}} \mathbf{z}_j\right)\right] = \left(E\left[e^{j\omega \mathbf{z}_j}\right]\right)^{\mathbf{J}} = \left[\Phi_{\mathbf{z}_j}(\omega)\right]^{\mathbf{J}}. \qquad \text{(A-10)}$$

Taking the expectation over **J** using (A-1) gives:

$$\Phi_{\mathbf{Z}_K}(\omega) = \sum_{n=0}^{\infty} \frac{e^{-K}K^n}{n!}\left[\Phi_{\mathbf{z}_j}(\omega)\right]^n = \exp\left\{K\left[\Phi_{\mathbf{z}_j}(\omega) - 1\right]\right\}. \qquad \text{(A-11)}$$

Thus, **Z** has a compound Poisson distribution.* Letting $\nu = 2/\gamma$, (A-4) gives the characteristic function of $\mathbf{z}_j$ as:

$$\Phi_{\mathbf{z}_j}(\omega) = \int_0^{\infty} f_{\mathbf{z}_j}(z) e^{j\omega z} dz = \frac{2}{\gamma K} \int_{K^{-\gamma/2}}^{\infty} z^{-1-\nu} e^{j\omega z} dz\,. \qquad \text{(A-12)}$$

The "second characteristic function" of $\mathbf{Z}_K$ is defined as the natural logarithm of the characteristic function.† Hence,

* See William Feller, *An Introduction to Probability Theory and Its Applications,* second ed., vol. II, New York: Wiley, 1971.

† See A. Papoulis, *Probability, Random Variables, and Stochastic Processes.* New York: McGraw-Hill, 1965.

$$\Psi_{\mathbf{Z}_K}(\omega) \triangleq \ln \Phi_{\mathbf{Z}_K}(\omega) = K\Big[\Phi_{\mathbf{Z}_K}(\omega) - 1\Big] = \frac{2}{\gamma} \int_{K^{-\gamma/2}}^{\infty} z^{-1-\nu}\, e^{j\omega z} dz - K \qquad \text{(A-13)}$$

Integrating by parts gives and recalling that $\mathbf{Z} = \lim_{K\to\infty} \mathbf{Z}_K$ gives:*

$$\Psi_{\mathbf{Z}}(\omega) = \lim_{K\to\infty} \Psi_{\mathbf{Z}_K}(\omega) = j\omega \int_0^{\infty} z^{-\nu}\, e^{j\omega z} dz$$

$$= -\,|\,\omega\,|^{\,\nu}\,\Gamma(1-\nu) e^{-j\pi\nu/2}\,, \quad \omega \geq 0$$

$$= -\,|\,\omega\,|^{\,\nu}\,\Gamma(1-\nu) e^{j\pi\nu/2}\,, \quad \omega < 0\,, \qquad \text{(A-14)}$$

where $\Gamma(\cdot)$ is the Gamma function.†

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* See also *Tables of Integral Transforms*, A. Erdelyi, *et al*, vol. 1, New York: McGraw-Hill, 1954, p. 10 §1.3, #1, and p. 68 §2.3 #1.

† See Abramowitz and Stegun, *op. cit.*, chapter 6.

C. The PDF and CDF of the Aggregate Interference

The pdf of **Z** is given by the Fourier inversion formula:

$$f_Z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_Z(\omega)e^{-j\omega z}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\Psi(\omega)}\, e^{-j\omega z}\, d\omega$$

$$= \frac{1}{2\pi}\sum_{k=0}^{\infty}\int_{-\infty}^{\infty} \frac{[\Psi(\omega)]^k}{k!}\, e^{-j\omega z}\, d\omega. \tag{A-15}$$

Letting $x \triangleq -\Gamma(1-\nu)e^{-j\pi\nu/2} = \Gamma(1-\nu)e^{j\pi(1-\nu/2)}$, $\Psi_Z(\omega) = \omega^\nu x$ for $\omega \geq 0$ and $\Psi_Z = -\omega^\nu x^*$ for $\omega < 0$ (where x^* denotes the complex conjugate of x), and (A-15) becomes:

$$f_Z(z) = \frac{1}{2\pi}\sum_{k=0}^{\infty}\frac{1}{k!}\int_0^{\infty}\left[(\omega^\nu x)^k e^{-j\omega z} + (\omega^\nu x^*)^k e^{j\omega z}\right]d\omega. \tag{A-16}$$

The integrals $\int_0^{\infty}\omega^{k\nu}e^{-j\omega z}d\omega$ and $\int_0^{\infty}\omega^{k\nu}e^{j\omega z}d\omega$ can be evaluated using a form of Euler's integral formula for the Gamma function*

$$\Gamma(y) = \xi^y\int_0^{\infty}\omega^{y-1}e^{-\omega\xi}\, d\omega\,,\quad \mathrm{Re}\{y\} > 0\,,\ \ \mathrm{Re}\{\xi\} > 0\,, \tag{A-17}$$

* Abramowitz and Stegun, *op. cit.*, p. 255.

where Re{·} denotes the real part of the complex argument and the condition on ξ is necessary to assure convergence of the integral.

Letting $\xi = z - j\varepsilon$, where z and ε are real and positive, (A-17) gives:

$$\int_0^\infty \omega^{k\nu} e^{-j\omega z} d\omega = \lim_{\varepsilon \to 0} \int_0^\infty \omega^{k\nu} e^{-j\omega\xi} d\omega = \lim_{\varepsilon \to 0} \frac{\Gamma(k\nu + 1)}{(j\xi)^{k\nu + 1}}$$

$$= \frac{\Gamma(k\nu + 1)}{(jz)^{k\nu + 1}} = \frac{\Gamma(k\nu + 1)}{z^{k\nu + 1} e^{-j\pi(k\nu + 1)/2}}, \quad z > 0, \qquad \text{(A-18a)}$$

and

$$\int_0^\infty \omega^{k\nu} e^{j\omega z} d\omega = \lim_{\varepsilon \to 0} \int_0^\infty \omega^{k\nu} e^{-j\omega\xi^*} d\omega = \lim_{\varepsilon \to 0} \frac{\Gamma(k\nu + 1)}{(j\xi^*)^{k\nu + 1}}$$

$$= \frac{\Gamma(k\nu + 1)}{(-jz)^{k\nu + 1}} = \frac{\Gamma(k\nu + 1)}{z^{k\nu + 1} e^{j\pi(k\nu + 1)/2}}, \quad z > 0, \qquad \text{(A-18b)}$$

Eq. (A-16) then becomes:

$$f_Z(z) = \frac{1}{2\pi} \sum_{k=0}^{\infty} \frac{\Gamma(k\nu + 1)}{k! z^{k\nu + 1}} \left[x^k e^{-j\pi(k\nu + 1)/2} + (x^*)^k e^{j\pi(k\nu + 1)/2} \right]$$

$$= \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{\Gamma(k\nu + 1)[\Gamma(1-\nu)]^k}{k! z^{k\nu + 1}} \cos\left[k\pi(1-\nu/2) - \pi(k\nu + 1)/2 \right]$$

$$= \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{\Gamma(k\nu + 1)[\Gamma(1-\nu)]^k}{k! z^{k\nu + 1}} \sin k\pi(1-\nu), \quad z > 0. \qquad \text{(A-19)}$$

The argument of the sum vanishes for $k = 0$, and (A-19) can be written as:*

$$f_Z(z) = \frac{1}{\pi z} \sum_{k=1}^{\infty} \frac{\Gamma(k\nu + 1)}{k!} \left[\frac{\Gamma(1-\nu)}{z^{\nu}}\right]^k \sin k\pi(1-\nu)\,, \quad z > 0. \qquad \text{(A-20)}$$

The CDF is then:

$$F_Z(z) = 1 - \int_z^{\infty} f_Z(\xi)d\xi = 1 - \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\Gamma(k\nu)}{k!} \left[\frac{\Gamma(1-\nu)}{z^{\nu}}\right]^k \sin k\pi(1-\nu)\,, z > 0. \text{(A-21)}$$

Note that for $z >> 1$, the first term in the series dominates. Since $\Gamma(\nu)\Gamma(1-\nu) = \pi \csc\pi(1-\nu)$† $F_Z(z) \simeq 1 - z^{-\nu}$ for $z >> 1$.

D. Closed-Form Expressions for Fourth-Power Propagation

For the special case of $\gamma = 4$ ($\nu = 1/2$), (A-20) and (A-21) can be reduced to closed form.‡ Since $\sin k\pi/2$ vanishes for even values of k, and $\Gamma(1/2) = \sqrt{\pi}$, (A-20) becomes:

$$f_Z(z) = \frac{1}{\pi z^{3/2}} \sum_{k=0}^{\infty} \frac{\Gamma(k + 3/2)\pi^{k+1/2}}{(2k + 1)!\, z^k} (-1)^k\,, \quad \gamma = 4\,, \quad z > 0. \qquad \text{(A-22)}$$

* Expressions equivalent to (A-20) and (A-21) are given by E. S. Sousa and J. A. Silvester in "Optimum Transmission Ranges in a Direct-Sequence Spread-Spectrum Multihop Packet Radio Network," *IEEE Journal on Selected Areas in Communications,* vol. 8, no. 5, June 1990, pp. 762-771. However, the expression given in that paper for the CDF is incorrect and actually represents the complementary distribution.

† Abramowitz and Stegun, *op. cit.*, p. 256, 6.1.17.

‡ This also is noted by Sousa and Silvester (*op. cit.*).

With the identity:*

$$\Gamma\left[2(k+1)\right] = \frac{\Gamma(k+1)\,\Gamma(k+3/2)\,z^{2k+3/2}}{\sqrt{2\pi}}, \qquad \text{(A-23)}$$

and the fact that $\Gamma[2(k+1)] = \Gamma(2k+2) = (2k+1)!$ (A-23) yields:

$$\frac{\Gamma(k+3/2)}{(2k+1)!} = \frac{\sqrt{2\pi}}{k!\,2^{2k+3/2}} = \frac{\sqrt{\pi}}{2\cdot k!\cdot 4^k}, \qquad \text{(A-24)}$$

and (A-22) is seen to be:

$$f_Z(z) = \frac{1}{2z^{-3/2}} \sum_{k=0}^{\infty} \frac{(-\pi/4z)^k}{k!}$$

$$= \frac{1}{2} z^{-3/2} e^{-\pi/4z}, \quad \gamma = 4. \qquad \text{(A-25)}$$

In a similar manner, (A-21) reduces for $\gamma = 4$ to:

$$F_Z(z) = 1 - \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{\Gamma(k+1/2)}{(2k+1)!} \frac{\pi^{k+1/2}}{z^{k+1/2}} (-1)^k, \quad \gamma = 4, \; z > 0. \qquad \text{(A-26)}$$

With the identity:†

* Abramowitz and Stegun, *op. cit.*, p. 256, 6.1.18.

† *Id.*

$$\Gamma(2k) = \frac{\Gamma(k)\,\Gamma(k + 1/2)2^{2k-1/2}}{\sqrt{2\pi}}\,, \tag{A-27}$$

and with $(2k + 1)! = (2k - 1)! \cdot (2k + 1) \cdot 2k$, and $\Gamma(2k) = (2K - 1)!$,

$$\frac{\Gamma(k + 1/2)}{(2k + 1)!} = \frac{2\sqrt{\pi}}{k!\,(2k + 1)\,2^{2k+1}}\,. \tag{A-28}$$

Substituting (A-28) into (A-26) yields:

$$F_Z(z) = 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\pi^{k+1/2}}{k!\,(2k + 1)\,z^{k+1/2}}\,(-1)^k$$

$$= 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\left(\sqrt{\pi}/2\sqrt{z}\right)^{2k+1}}{k!\,(2k + 1)}\,(-1)^k$$

$$= \mathrm{erfc}\left(\frac{\sqrt{\pi}}{2\sqrt{z}}\right), \quad \gamma = 4\,, \tag{A-29}$$

where erfc(·) is the complementary error function* defined as

$$\mathrm{erfc}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2}\, d\xi = 1 - \mathrm{erf}(x)\,, \tag{A-30a}$$

* Abramowitz and Stegun, *op. cit.*, chapter 7.

and erf(x) is the error function

$$\mathrm{erf}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{x^{2k+1}(-1)^k}{k!\,(2k+1)} . \tag{A-30b}$$

E. Expressions for the Blocking Probability

From (A-8) and (A-21), the blocking probability is:

$$P_b(\alpha) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\Gamma(2k/\gamma)}{k!} \left[\alpha\Gamma(1-2/\gamma)\right]^k \sin k\pi(1-2/\gamma), \quad \gamma > 2. \tag{A-31}$$

As noted previously, $F_Z \simeq 1 - z^{-2/\gamma}$ for $z >> 1$. Hence, from (A-8), $P_b(\alpha) \simeq \alpha$ for $\alpha << 1$. For $\gamma = 4$, (A-8) and (A-29) give:

$$P_b(\alpha) = \mathrm{erf}\left(\frac{\alpha\sqrt{\pi}}{2}\right) \quad \text{for } \gamma = 4. \tag{A-32}$$

F. The Single-Interferer Case

In the context of this model, the blocking probability for the "single-interferer" case is easily derived by recalling that the average interference source density is $1/\pi$ active transmitters per unit area, and the normalized interference power from a source a distance s from the receiver is $s^{-\gamma}$. Since the number of active transmitters within (normalized) distance s of the receiver is a Poisson-distributed random variable with mean value s^2, the probability that there are *no* active transmitters within that distance of the receiver is e^{-s^2}. Thus, since the normalized interference from a single source at distance s is $\mathbf{Z} = s^{-\gamma}$, the $P\{\mathbf{Z} < z\}$ for the "single-interferer" case is equal to the probability that there are no interfering transmitters within distance $s = z^{-1/\gamma}$ of the receiver. Hence, for the "single interferer" case,

$$F_Z(z) = \exp\left(-z^{-2/\gamma}\right), \quad z \geq 0, \tag{A-33}$$

and from (A-8), the blocking probability for the single-interference case at the edge of the service area is:

$$P_b(\alpha) = 1 - e^{-\alpha}. \tag{A-34}$$

Fig. A-1 shows $F_Z(z)$ for $\gamma = 3.0$, 3.5, and 4.0, for both the multiple-interferer and single-interferer cases, and the blocking probability is shown in Fig. 6 in the body of this paper.

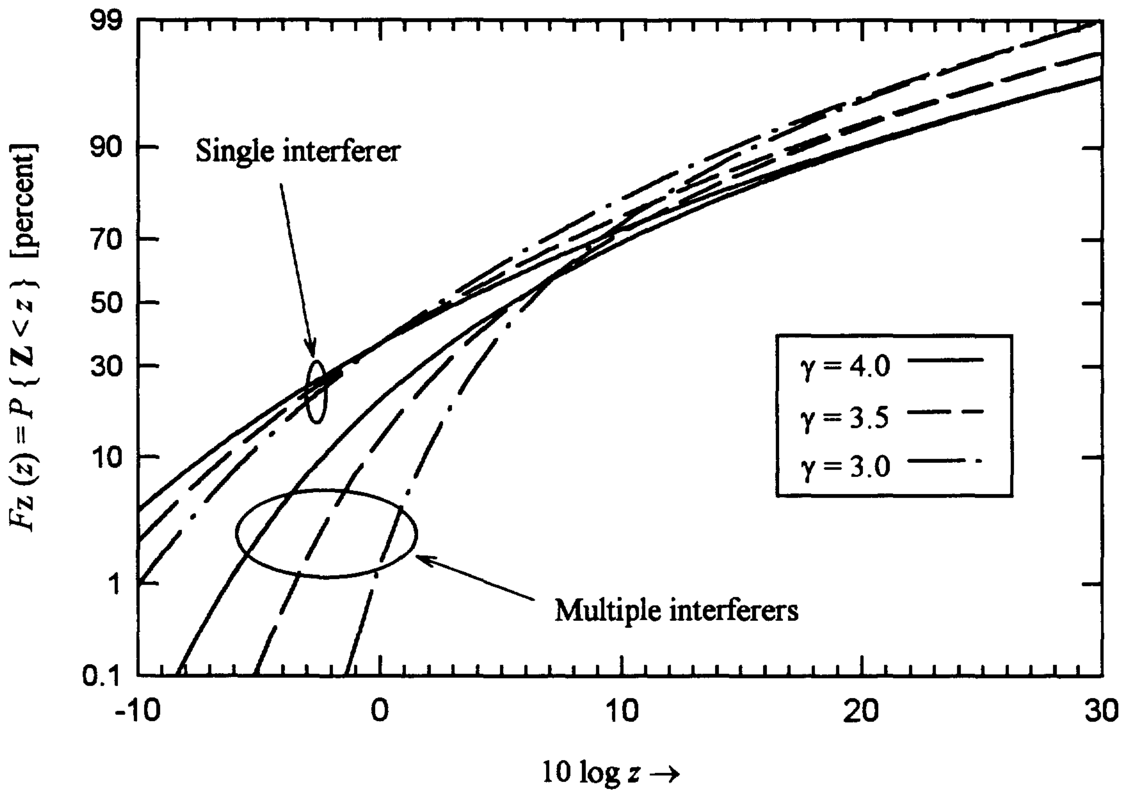

Figure A-1. *The CDF of the normalized aggregate interference power.*